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THE STABILITY OF GAS-FLOW OVER A STAGNANT LIQUID IN A PIPE AND THE ONSET OF SLUGGING

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Abstract—A theoretical and experimental study is made of the stability of a cocurrent gas–liquid flow on the curved portion of a pipe traversing hilly terrain. For small values of the dimensionless stability parameter, experimental agreement with inviscid theory is excellent. For larger values, ripples and friction account for the discrepancy between theory and observation.

1. INTRODUCTION

The simultaneous transport of gas and liquid occurs in many industrial situations, such as during the removal of crude oil and gas from offshore wells. Stratified flow and slug-flow are the likely regimes to be found in these applications. Many authors have presented their studies on two-phase flow in horizontal and inclined pipes. However the pipe profile is usually a curved line with a sequence of ascending and descending sections following the slopes of the sea-bed or the ground where it is laid. To our knowledge, no attention has been paid to the phenomenon which takes place in the curved parts of such pipes. The lowest portions are the source of unstable flow for high gas–liquid flow-rate ratio and control the formation of long bubbles for a moderate gas–liquid flow-rate ratio when slug-flow occurs in the following ascending duct. We present here a study of the onset of unstable flow in the low part of a pipe owing to the deposition or condensation of liquid which accumulates at the bottom.

2. DESCRIPTION OF THE INSTABILITY MODEL

Due to deposition or condensation in a long curved pipe, some liquid will accumulate by gravity in the lowest part of the pipe. This trapped volume will slowly increase until a slugging condition is reached and the flow is no longer steady.

From the reduction in area due to the trapped volume, the velocity increases and the pressure decreases over the pocket of liquid therefore causing suction of the free surface. This suction is balanced by hydrostatic pressure. For a moderate gas flow rate, the free surface is in equilibrium and its profile is a hump shape. If the gas-flow or trapped volume is increased, a critical value is reached where equilibrium is impossible and bridging occurs; a lump of liquid fills the whole pipe cross section and is blown uphill by the pressure rise.

In view of the practical applications, the following restrictions are used in developing the theory of this phenomenon:

Fully developed incompressible turbulent flow of the gas is the regime in the pipe. Calculations are made using a flat velocity profile and friction is neglected.

The trapped volume of liquid increases slowly with time so that the assumption of steady-state holds.

The radius of curvature of the bend in the vertical plane is much larger than the pipe diameter (it is in practice greater than 200 diameters). Surface tension effects are neglected.

The exact profile of the pipe is irrelevant and can be described in the vicinity of the point of minimum altitude by its radius of curvature R only. The flow of gas is a one dimensional flow in a slowly varying area duct and the pressure is calculable by Bernoulli's law.

Figure 1 shows the variables used in the description of the flow and the free surface profile. The void fraction at any cross-section is α, the elevation of the free surface is h, and the height of

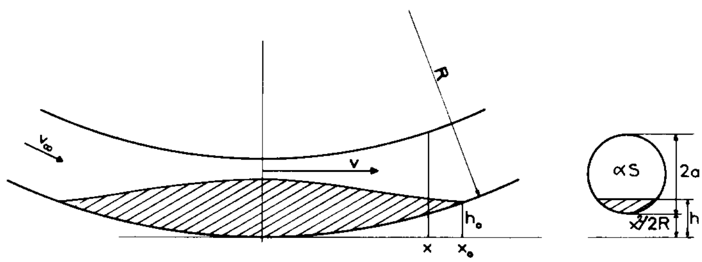

Figure 1. Sketch of the gas flow over a pocket of liquid.

the bottom of the pipe is $x^2/2R$ at a distance x from the lowest point. Far upstream, the velocity is v_∞ and the pressure p_∞. The gas and liquid density are ρ_G and ρ_L respectively. For a given cross-section, α is a function of $\eta = h - x^2/2R$ alone. Let $2a$ be the height or the diameter of the channel, and use the dimensionless variables

$$\phi = \frac{\rho_G}{\rho_L - \rho_G} \cdot \frac{v_\infty^{\;2}}{2ga}, \qquad [1]$$

$$h^+ = \frac{h}{a}, \qquad [2]$$

$$x^+ = \sqrt{\left(\frac{x^2}{2Ra}\right)}, \qquad [3]$$

$$\eta^+ = h^+ - x^{+2}, \quad 0 \leq \eta^+ \leq 2. \qquad [4]$$

The pocket of liquid, of volume ν extends from $-x$ to $+x$ and we have

$$h_0^+ = x_0^{+2}(\eta_0^+ = 0), \qquad [5]$$

$$\nu^+ = \frac{\nu}{2S\sqrt{(2Ra)}} = \int_0^{x_0^+} (1-\alpha)\,\mathrm{d}x^+ \qquad [6a]$$

where S is the cross-section area of the pipe; α is a known function of

$$\eta^+, \alpha = f(\eta^+). \qquad [7a]$$

For instance, for a rectangular channel we have

$$\alpha = 1 - \frac{\eta^+}{2}, \qquad [8]$$

and for a circular cross section,

$$\alpha = 1 - \frac{\cos^{-1}(1-\eta^+) - (1-\eta^+)\sqrt{(1-(1-\eta^+)^2)}}{\pi}. \qquad [9]$$

The free surface equilibrium condition is found by equating the pressure difference computed from Bernoulli's theorem between an upstream section and a section at abscissa x, $(-x_0 < x < x_0)$ to the hydrostatic pressure difference, i.e.

$$h^+ - h_0^+ = \phi\left(\frac{1}{\alpha^2} - 1\right) \qquad [10]$$

in dimensionless form.

Solving this equation simultaneously with

$$\alpha = f(h^+ - x^{+2}) \qquad [7b]$$

gives the free-surface profile as a function of h_∞^+

$$h^+ = h^+(x^{+2}, \phi, h_0^+)$$

showing symmetry about the mid-point $x = 0$.

An alternative is to solve simultaneously [10],

$$\alpha = f(h^+ - x^{+2}), \qquad [7c]$$

and

$$\nu^+ = \int_0^{\sqrt{(h_0^+)}} (1-\alpha)\,\mathrm{d}x^+, \qquad [6b]$$

to obtain a solution as a function of the volume

$$h^+ = h^+(x^{+2}, \phi, \nu^+).$$

It can be shown (see the appendix) that the system [7], [10] or [6], [7], [10] has two roots. One corresponds to stable equilibrium for any value of x in the interval $-x_0 < x < x_0$ for $\phi < \phi^*$, where ϕ^* is a value to be specified later. For $\phi = \phi^*$, there is one stable solution for $x \neq 0$, and a double root for $x = 0$. In the latter case, the profile has an angular point and the solution is unstable. For $\phi > \phi^*$ there is no solution in an interval $[-x_1, +x_1]$ and therefore no stable flow can exist under these conditions.

The value of ϕ^* is given by eliminating α and η^+ between

$$\phi = -\frac{\alpha^3}{2(\mathrm{d}\alpha/\mathrm{d}\eta^+)} \qquad [11]$$

obtained by differentiating [10] with respect to η^+,

$$\eta^+ - h^+ = \phi\left(\frac{1}{\alpha^2} - 1\right) \quad \text{([10] written for } x^+ = 0\text{)}, \qquad [10']$$

and [7a].

We thus obtain a value $\phi^*(h^+)$, or $\phi^*(\nu^+)$ if we use the relation [6] to relate h_0^+ and ν^+. The predicted shapes of the free-surface profile are shown on figure 2 for different values of ϕ.

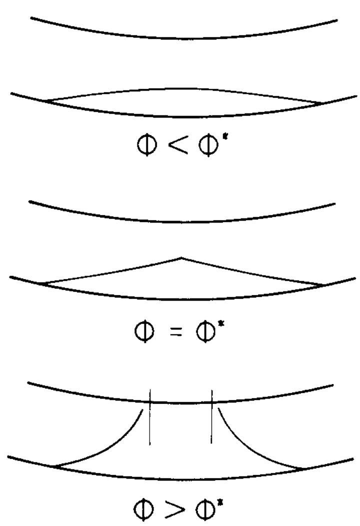

Figure 2. Predicted shapes of the free surface for different values of ϕ.

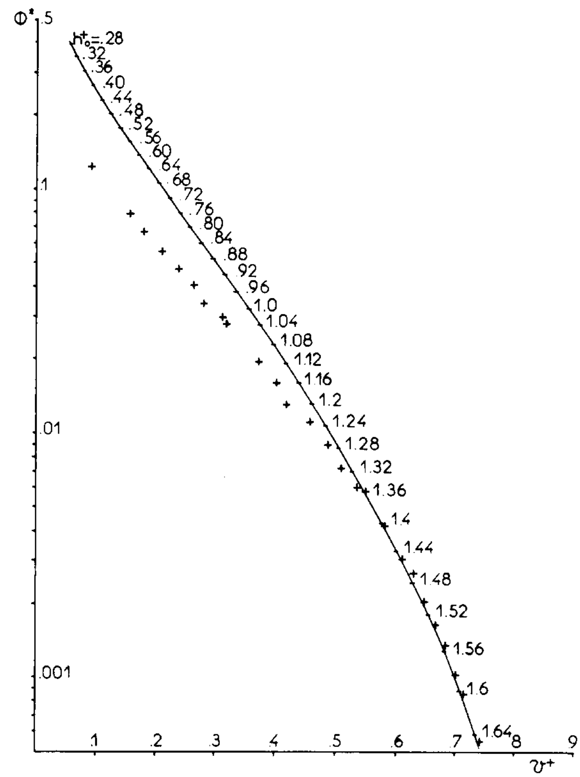

Figure 3. Comparison of the solution of system [6], [7], [10] with experimental results (+).

The values of ϕ^* as a function of ν^+ and h_0^+ are given for a circular cross-section in figure 3 (solid curve).

For a rectangular cross-section, the values of ϕ^* and h_0^+ are related by

$$1-\frac{h_0^+}{2}=\frac{3\phi^{*1/3}-\phi^*}{2}. \qquad [12]$$

3. EXPERIMENTAL RESULTS AND DISCUSSION

A polyvinyl chloride (PVC) hose of inside diameter 47 mm was laid on a frame and given a radius of curvature of 175 m in a vertical plane. The air flow was regulated through a sonic nozzle and controled volumes of water could be introduced in the lowest part of the tube. The volume was increased very slowly until bridging occured. The air rate was varied between the low Reynolds number limit of 2400 and the upper limit where the pocket of water would creep up the slope under friction.

It can be seen from the plot of the results (figure 3, dotted curve) that the agreement with inviscid theory is excellent for small values of $\phi^*(\phi^* < 10^{-2})$. At higher values, the necessary amount of flow to obtain the instability is somewhat less than that predicted by theory (of the order of 25% for $\phi^* = 0.1$). No slugging could be obtained for ϕ^* greater than 0.13, the water being entrained up the slope by friction. This latter phenomenon is thought to be dependent upon the angle of the ascending part of the tube, the slope of which was in our case 0.02.

Ripples and friction account for the discrepancy between the theory and the observed values. For instance, when $\nu^+ = 0.3$, the calculated values of h^+ and $h^+(0)$ are $h^+ = 0.9$ and $h^+(0) = 1.1$. The observed value of the ripple height is of the order of 5 mm. Thus an instantaneous value of h^+ at $x = 0$ could be increased by $\Delta h^+ \simeq 0.2$. Taking $h^+(0) = 1.1 + 0.2 = 1.3$ as a basis, the corresponding value of ϕ^* would be 0.028 instead of 0.05 for a smooth interface, a value close to the experimental value.

Friction on the rough interface causes the whole pocket of liquid to be shifted downstream and bridging occurs at a distance from the centre line which can reach 1 m for large values of ϕ^*. Assuming a constant friction pressure drop, both the displacement of the observed bridging point and the reduction in critical flow rate agree to give a value of the friction factor of the order of

$$\left|\frac{\Delta p}{\Delta x}\right| \cdot \frac{2a}{1/2\rho_G v_\infty^2} \simeq 10^{-1} \text{ for } \phi^* = 0.1.$$

As the system is very sensitive to perturbation when the point of operation is near the instability curve, it is possible once slugging has started, to reduce the air rate without returning to stable flow, therefore obtaining oscillations in the stable region.

4. THE WALLIS-DOBSON EXPERIMENT

The instability model can be applied to a horizontal channel of rectangular cross-section. Our theory gave the result [12].

However, if a stagnation point is present, as seems to be the case in one of the experiments of the authors, the reference pressure and level can be taken at this point and [10] would be modified to

$$h^+ - h_0^+ = \frac{\phi}{\alpha^2}, \qquad [13]$$

and the stability condition would then be

$$\phi^* < \left(\frac{2}{3}\right)^3 \left(1 - \frac{h_0^+}{2}\right)^3. \qquad [14]$$

Equations [12] and [14], and Wallis & Dobson's 1973 results have been plotted on the same graph (figure 4) with the Wallis notation j_G^*, α, and it can be seen that the differences are quite small. The experimental values of ϕ^* tends to be lower than the inviscid theoretical ones, the larger the value of ϕ^* so that the results of Wallis & Dobson can very well be accounted for by [12] plus the effect of ripples and friction.

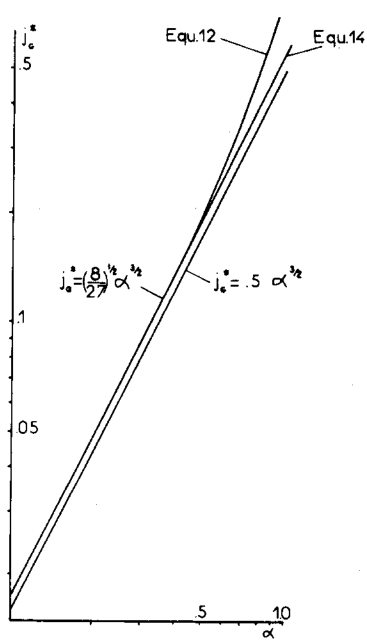

Figure 4. Comparison of [12] and [14] and Wallis & Dobson's (1973).

5. CONCLUSION

The results presented here have already been found useful in the prediction of slugging in gas-pipes laid on hilly terrain and experimentation is continuing on the behaviour of the oscillations, once unsteady flow has started.

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APPENDIX

Figure 5 shows a graphical solution for the roots of the system [7], [10]. The points S and S' are the stable roots for $x^+ = 0$ and $x^+ \neq 0$ respectively for $\phi < \phi^*$. The roots given by the intersection I or I′ are unstable solutions: for instance, let the free surface be represented by the

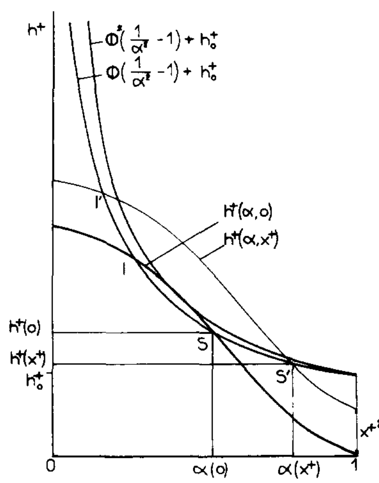

Figure 5. Graphical solution for the roots of system [7], [10].

equilibrium point I and give it a small increase of elevation, the suction $\phi^*[(1/\alpha^2) - 1]$ becomes larger than the gravity force $h^+ - h_0^+$, so that the resultant force is in the direction of the displacement; the equilibrium is therefore unstable. The converse is true for the points S and S' where the slopes are in the reverse order so that the force is a restoring force for any small displacement of the free surface.